M@thEm@ti¢§ 9

Real numbers

Part 3 (Powers)

Definition

a is an integer and $a \neq 0$ and n is a non zero natural number.

$$a^n = a \times a \times a \times \cdots \times a \qquad (n \text{ times})$$

By convention $a^0 = 1$

Example:

$$3^1 = 3 \quad ; \quad 3^2 = 3 \times 3 = 9 \quad ; \quad 3^3 = 3 \times 3 \times 3 = 27$$

$$\left(-\frac{1}{2}\right)^5 = \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) = -\frac{1}{32}$$

Power of a negative exponent

a is a number other than zero and n is a natural greater than zero

❖ $a^{-n} = \frac{1}{a^n}$

Example:

$2^{-1} = \frac{1}{2}$; $2^{-2} = \frac{1}{2^2} = \frac{1}{4}$; $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$

$-3^{-3} = -\frac{1}{3^3} = -\frac{1}{27}$; $\left(\frac{5}{4}\right)^{-2} = \left(\frac{4}{5}\right)^2$

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Sign of an exponent

Let $a \in \mathbb{R}$ *and* $n \in \mathbb{N}$ *where* $a \neq 0$ and $n > 0$

❖ If :

- $a > 0$, then $a^n > 0$
- $a < 0$, and $\begin{cases} n \text{ is even, then } a^n > 0 \\ n \text{ is odd, then } a^n < 0 \end{cases}$

Example:

3^5 is positive since the base 3 is positive.

$(-4)^6$ is positive since the exponent is 6 which is an even number.

$(-4)^5$ is negative since the exponent is 5 which is odd

Operations on powers

a and b are numbers other than zero and m & n are two integers.

- $a^m \times a^n = a^{m+n}$
- $\dfrac{a^m}{a^n} = a^{m-n} = \dfrac{1}{a^{n-m}}$
- $(a^m)^n = a^{m \times n}$
- $(a \times b)^m = a^m \times b^m$
- $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$

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Operations on powers

Example:

$$2^5 \times 2^3 = 2^{5+3} = 2^8$$

$$3^{11} \times 3^{-12} = 3^{11+(-12)} = 3^{-1}$$

$$5^{14} \div 5^8 = 5^{14-8} = 5^6$$

$$7^{-10} \div 7^{-8} = 7^{-10-(-8)} = 7^{-2}$$

$$(3^5)^2 = 3^{5\times 2} = 3^{10}$$

$$\left((-3)^{-2}\right)^5 = (-3)^{-2\times 5} = (-3)^{10}$$

$$(4 \times 3)^2 = 4^2 \times 3^2 = 16 \times 9 = 144$$

Operations on powers

a and b are numbers other than zero and m & n are two integers.

IMPORTANT

$(a \pm b)^n \neq a^n \pm b^n$

Example:

$3^2 + 2^2 = 9 + 4 = 13$ but $(3+2)^2 = 5^2 = 25 \neq 13$

$5^3 - 4^3 = 125 - 64 = 61$ but $(5-4)^3 = 1^3 = 1 \neq 61$

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Operations on powers

IMPORTANT

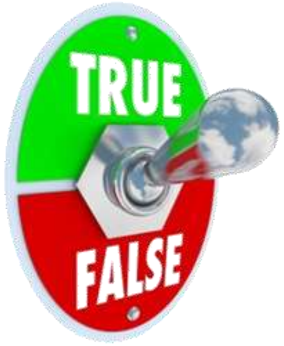
A decimal number can be written in form of $a \times 10^{-n}$ where a is an integer and n is number of digits after the decimal point.

Example:

$2.5 = 25 \times 10^{-1}$; $1.25 = 125 \times 10^{-2}$

$0.0000256 = 256 \times 10^{-7}$

This form of the number is easier to use in simplification.

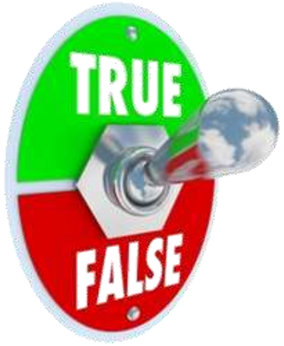


Answer with true or false

Statement	True	False
$-3^0 = 1$		X

Justification:

$$-3^0 = -1 \neq 1$$

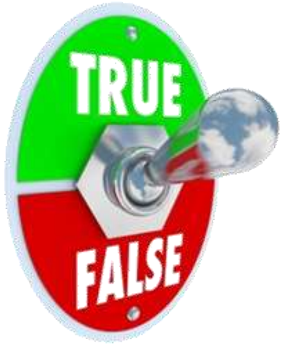


Answer with true or false

Statement	True	False
$2^3 + 2^3 = 4^6$		X

Justification:

$$2^3 + 2^3 = 2^3(1+1) = 2^3 \times 2 = 2^3 \times 2^1 = 2^{3+1} = 2^4$$

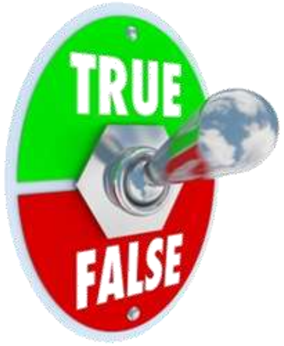


Answer with true or false

Statement	True	False
The triple of 3^3 is 3^4	X	

Justification:

Triple of 3^3 is $3 \times 3^3 = 3^1 \times 3^3 = 3^{1+3} = 3^4$

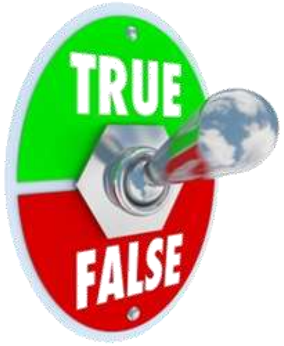


Answer with true or false

Statement	True	False
The inverse of 10^4 is 0.0001	X	

Justification:

The inverse of 10^4 is $\frac{1}{10^4} = 0.0001$

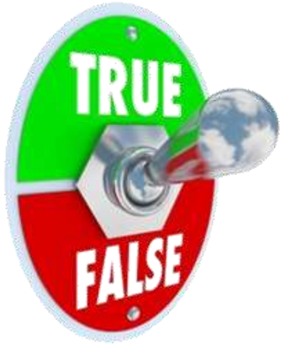


Answer with true or false

Statement	True	False
$-(5+120 \div 9)^0 - (-3)^2 = 8$		X

Justification:

$$-(5+120 \div 9)^0 - (-3)^2 = -1 - 9 = -10$$

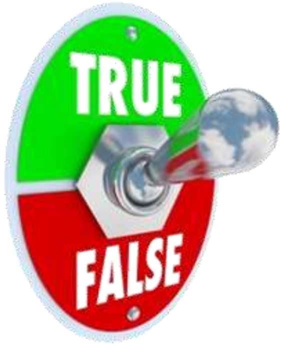


Answer with true or false

Statement	True	False
$5^4 \times 2^4 \times (10^5)^3 = 10^{12}$	X	

Justification:

$$\begin{aligned} 5^4 \times 2^4 \times (10^5)^3 &= (5 \times 2)^4 \times 10^{15} \\ &= 10^4 \times 10^{15} \\ &= 10^{4+15} \\ &= 10^{19} \end{aligned}$$



Answer with true or false

Statement	True	False
$\frac{(10^5)^{-2}\times 10^6}{0.001} = 10^2$		X

Justification:

$$\frac{(10^5)^{-2}\times 10^6}{0.001} = \frac{10^{-10}\times 10^6}{10^{-3}}$$

$$= \frac{10^{-10+6}}{10^{-3}}$$

$$= \frac{10^{-4}}{10^{-3}}$$

$$= 10^{-4-(-3)} = 10^{-1}$$